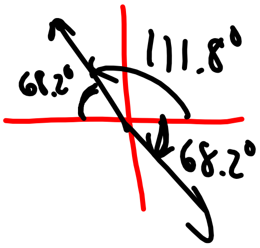
45. $-2i + 5j$

$\theta = \tan^{-1} \frac{5}{-2}$

$\theta = -68.2$



80

0 – 8 +

9 – 16 ✓

17↑ –

8-3 Study Guide and Intervention
Dot Products and Vector Projections

Dot Product The dot product of $\mathbf{a} = \langle a_1, a_2 \rangle$ and $\mathbf{b} = \langle b_1, b_2 \rangle$ is defined as $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2$.

KeyConcept Orthogonal Vectors

The vectors **a** and **b** are orthogonal if and only if $\mathbf{a} \cdot \mathbf{b} = 0$.

KeyConcept Properties of the Dot Product

If **u**, **v**, and **w** are vectors and k is a scalar, then the following properties hold.

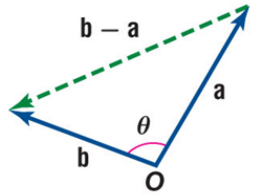
Commutative Property	$\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$
Distributive Property	$\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$
Scalar Multiplication Property	$k(\mathbf{u} \cdot \mathbf{v}) = k\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot k\mathbf{v}$
Zero Vector Dot Product Property	$\mathbf{0} \cdot \mathbf{u} = 0$
Dot Product and Vector Magnitude Relationship	$\mathbf{u} \cdot \mathbf{u} = \lvert\mathbf{u}\rvert^2$

$\frac{1}{2}(4 \cdot 5)$

KeyConcept Angle Between Two Vectors

If θ is the angle between nonzero vectors **a** and **b**, then

$$\cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|\,|\mathbf{b}|}.$$



Proof

Consider the triangle determined by **a**, **b**, and **b** − **a** in the figure above.

$\|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 - 2\,\|\mathbf{a}\|\,\|\mathbf{b}\|\cos\theta = \|\mathbf{b} - \mathbf{a}\|^2$	Law of Cosines
$\|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 - 2\,\|\mathbf{a}\|\,\|\mathbf{b}\|\cos\theta = (\mathbf{b} - \mathbf{a}) \cdot (\mathbf{b} - \mathbf{a})$	$\|\mathbf{u}\|^2 = \mathbf{u} \cdot \mathbf{u}$
$\|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 - 2\,\|\mathbf{a}\|\,\|\mathbf{b}\|\cos\theta = \mathbf{b} \cdot \mathbf{b} - \mathbf{b} \cdot \mathbf{a} - \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{a}$	Distributive Property for Dot Products
$\|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 - 2\,\|\mathbf{a}\|\,\|\mathbf{b}\|\cos\theta = \|\mathbf{b}\|^2 - 2\mathbf{a} \cdot \mathbf{b} + \|\mathbf{a}\|^2$	$\mathbf{u} \cdot \mathbf{u} = \|\mathbf{u}\|^2$
$-2\,\|\mathbf{a}\|\,\|\mathbf{b}\|\cos\theta = -2\mathbf{a} \cdot \mathbf{b}$	Subtract $\|\mathbf{a}\|^2 + \|\mathbf{b}\|^2$ from each side.
$\cos\theta = \dfrac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\|\,\|\mathbf{b}\|}$	Divide each side by $-2\|\mathbf{a}\|\,\|\mathbf{b}\|$.

Example 1: Find the dot product of u and v. Then determine if u and v are orthogonal.

a. $\mathbf{u} = \langle 5, 1 \rangle$, $\mathbf{v} = \langle -3, 15 \rangle$

$\mathbf{u} \cdot \mathbf{v} = 5(-3) + 1(15)$
$= 0$

Since $\mathbf{u} \cdot \mathbf{v} = 0$, **u** and **v** are orthogonal.

b. $\mathbf{u} = \langle 4, 5 \rangle$, $\mathbf{v} = \langle 8, -6 \rangle$

$\mathbf{u} \cdot \mathbf{v} = 4(8) + 5(-6)$
$= 2$

Since $\mathbf{u} \cdot \mathbf{v} \neq 0$, **u** and **v** are not orthogonal.

Example 2: Find the angle θ between vectors u and v if $\mathbf{u} = \langle 5, 1 \rangle$ and $\mathbf{v} = \langle -2, 3 \rangle$.

$\cos\theta = \frac{u \cdot v}{\lvert u \rvert \, \lvert v \rvert}$	Angle between two vectors
$\cos\theta = \frac{\langle 5,1 \rangle \cdot \langle -2,3 \rangle}{\lvert\langle 5,1 \rangle\rvert \, \lvert\langle -2,3 \rangle\rvert}$	**u** = ⟨5, 1⟩ and **v** = ⟨–2, 3⟩
$\cos\theta = \frac{-10+3}{\sqrt{26}\,\sqrt{13}}$	Evaluate.
$\theta = \cos -1 \frac{-10+3}{\sqrt{26}\,\sqrt{13}}$ or about $112°$	Simplify and solve for θ.

The measure of the angle between **u** and **v** is about $112°$.

Exercises

Find the dot product of u and v. Then determine if u and v are orthogonal.

1. **u** = ⟨2, 4⟩, **v** = ⟨–12, 6⟩

$\vec{u}\cdot\vec{v} = (2)(-12)+(4)(6)$

$= 0$

orthogonal

2. **u** = –8**i** + 5**j**, **v** = 3**i** –6**j**

$\vec{u}\cdot\vec{v} = (-8)(3)+(5)(-6)$

$= -54$

not orthogonal

Use the dot product to find the magnitude of the given vector.

3. **a** = ⟨9, 3⟩

$\vec{a}\cdot\vec{a} = (9)(9)+(3)(3)$

$\sqrt{|\vec{a}|^2} = \sqrt{90}$

$|\vec{a}| = 3\sqrt{10}$

4. **c** = ⟨–12, 4⟩

Find the angle θ between u and v to the nearest tenth of a degree.

5. **u** = ⟨–3, –5⟩, **v** = ⟨7, 12⟩

$$\cos\theta = \frac{(-3)(7)+(-5)(12)}{\sqrt{(-3)^2+(-5)^2}\cdot\sqrt{(7)^2+(12)^2}}$$

$$= \frac{-81}{(5.8)(13.9)}$$

$\cos^{-1}(\cos\theta) = \cos^{-1}(-1)$

$\theta = 180°$

6. **u** = 13**i** – 5**j**, **v** = 6**i** + 2**j**

HW: p. 506
1, 5, 11, 15, 17, 19, 21, 75-85 odd