KeyConcept Quick Tests for Symmetry in Polar Graphs

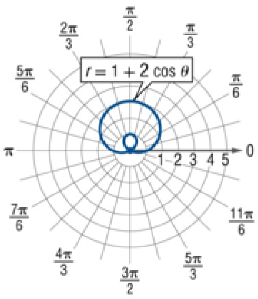
Words The graph of a polar equation is symmetric with respect to

- the polar axis if it is a function of $\cos \theta$, and
- the line $\theta = \frac{\pi}{2}$ if it is a function of $\sin \theta$.

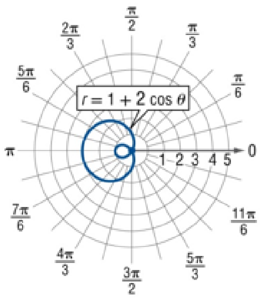
Example The graph of $r = 3 + \sin \theta$ is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

Use symmetry to graph $r = 1 + 2\cos\theta$.

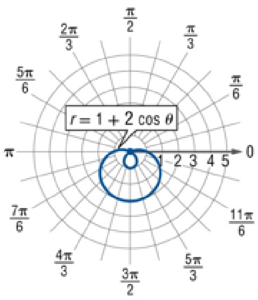
A.



C.



B.



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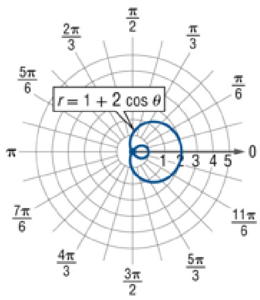


9-2 Study Guide and Intervention

Graphs of Polar Equations

Graphs of Polar Equations A **polar graph** is the set of all points with coordinates (r, θ) that satisfy a given polar equation. The position and shape of polar graphs can be altered by multiplying or adding to either the function or θ.

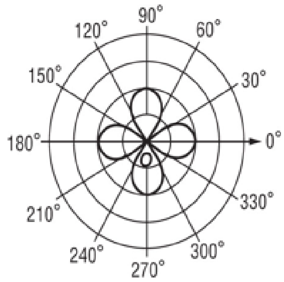
Example 1: Graph the polar equation $r = 2 \cos 2\theta$.

Make a table of values on the interval $[0, 2\pi]$.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$r = 2 \cos 2\theta$	2	1	0	1	–2	–1	0	1	2	1	0	–1	–2	–1	0	1	2

Graph the ordered pairs (r, θ) and connect with a smooth curve.

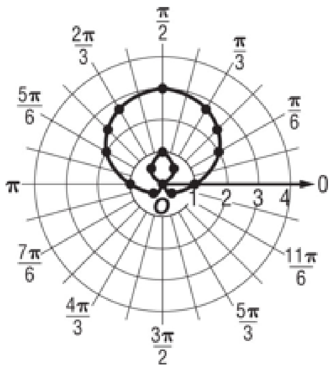
This type of curve is called a **rose**. Notice that the farthest points are 2 units from the pole and the rose has 4 petals.



Example 2: Graph the polar equation $r = 1 + 2 \sin \theta$. Round each r-value to the nearest tenth.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$r = 1 + 2 \sin \theta$	1	2	2.4	2.7	3	2.7	2.4	2	1	0	–0.4	–0.7	–1	–0.7	–0.4	0	1

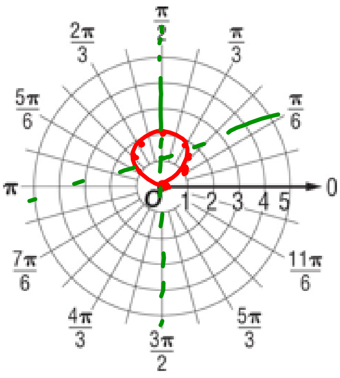
Graph the ordered pairs and connect them with a smooth curve. This type of curve is called a **limaçon**.



Exercises

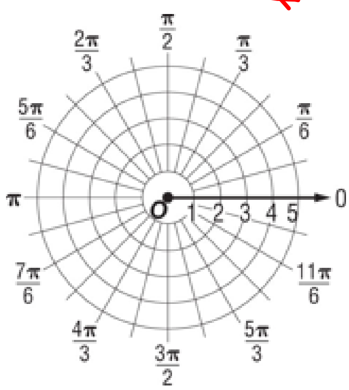
Graph each equation by plotting points.

1. $r = 2\sin\theta$

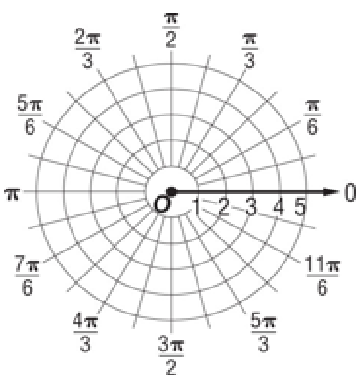


θ	r
0	0
$\frac{\pi}{6}$	1
$\frac{\pi}{4}$	$\sqrt{2} \approx 1.4$
$\frac{\pi}{3}$	$\sqrt{3} \approx 1.7$
$\frac{\pi}{2}$	2
$\frac{2\pi}{3}$	$\sqrt{3} = 1.7$
$\frac{3\pi}{4}$	$\sqrt{2} \approx 1.4$
$\frac{5\pi}{6}$	1
π	0
$\frac{7\pi}{6}$	-1
$\frac{5\pi}{4}$	$-\sqrt{2} = -1.4$
$\frac{4\pi}{3}$	$-\sqrt{3} = -1.7$
$\frac{3\pi}{2}$	$= -2$
$\frac{5\pi}{3}$	$-\sqrt{3} = -1.7$
$\frac{7\pi}{4}$	$-\sqrt{2} = -1.4$
$\frac{11\pi}{6}$	-1
2π	0

2. $r = 2 + 2\sin\theta$ $\quad -\frac{8\sqrt{2}}{2}$



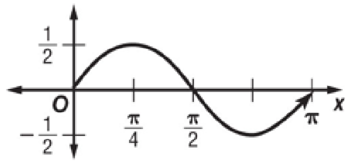
3. $r = 1 - 3\cos\theta$



Classic Polar Curves The graph of a polar equation is symmetric with respect to the polar axis if it is a function of $\cos\theta$, and to the line $\theta = \frac{\pi}{2}$ if it is a function of $\sin\theta$. It is symmetric to the pole if replacing (r, θ) with $(-r, \theta)$ or $(r, \pi + \theta)$ produces an equivalent equation. Knowing whether a graph is symmetric can reduce the number of points needed to sketch it.

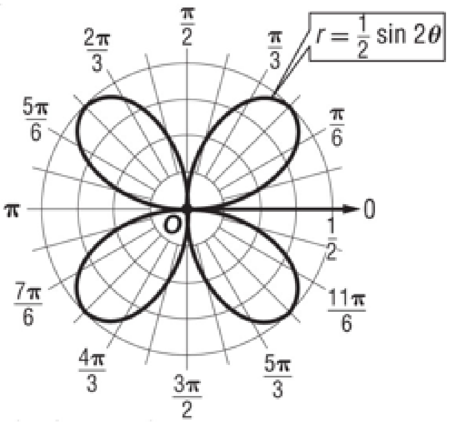
Example: Determine the symmetry, zeros, and maximum r-values of $r = \frac{1}{2}\sin 2\theta$. Then use this information to graph the function.

The function is symmetric with respect to the line $\theta = \frac{\pi}{2}$, so you can find points on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and then use line symmetry to complete the graph. To find the zeros and the maximum r-value, sketch the graph of the rectangular function $y = \frac{1}{2}\sin 2x$.



From the graph, you can see that $|y| = \frac{1}{2}$ when $x = \frac{\pi}{4}$, and $\frac{3\pi}{4}$ and $y = 0$ when $x = 0, \frac{\pi}{2}$, and π. That means that $|r|$ has a maximum value of $\frac{1}{2}$ when $\theta = \frac{\pi}{4}$ or $\frac{3\pi}{4}$ and $r = 0$ when $\theta = 0, \frac{\pi}{2}$, or π. Use these and a few additional points to sketch the graph of the function.

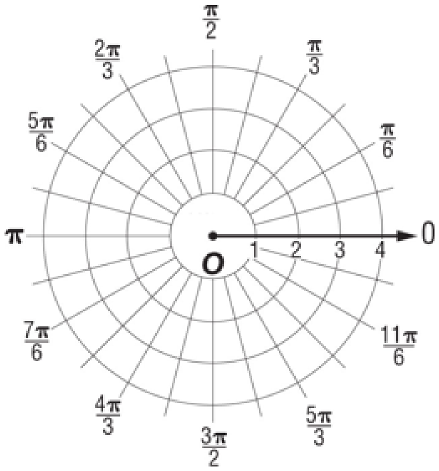
Use the axis of symmetry to complete the graph after plotting points on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.



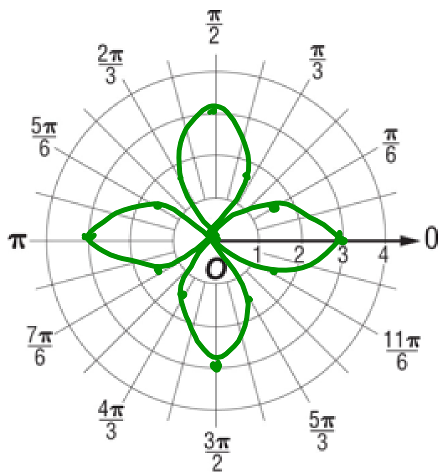
Exercises

Use symmetry, zeros, and maximum r-values to graph each function.

1. $r = 4 \sin 3\theta$



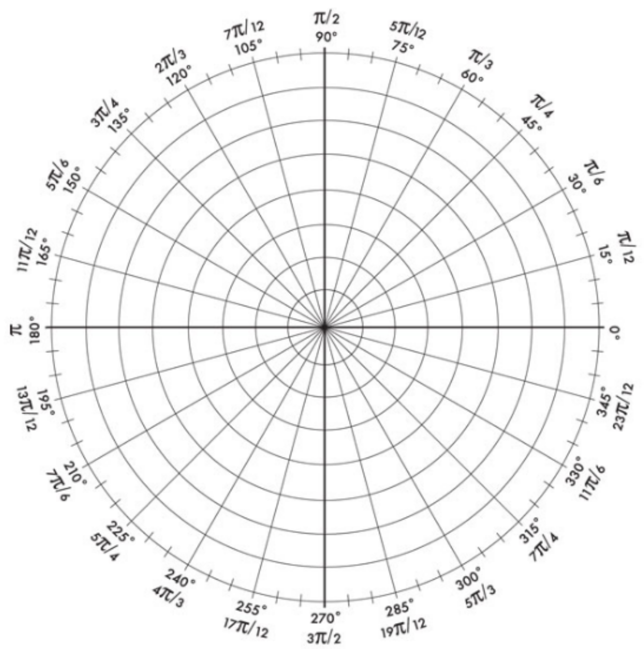
2. $r = 3 \cos 2\theta$



θ	r
0	3
$\frac{\pi}{6}$	1.5
$\frac{\pi}{4}$	0
$\frac{\pi}{3}$	-1.5
$\frac{\pi}{2}$	-3
$\frac{2\pi}{3}$	-1.5
$\frac{3\pi}{4}$	0
$\frac{5\pi}{6}$	1.5
π	3

HW: p. 548

1, 5, 9, 15, 19, 23, 27, 33, 35, 37


π/2
90°
5π/12
75°
π/3
60°
π/4
45°
π/6
30°
π/12
15°
0°
345°
23π/12
330°
11π/6
315°
7π/4
300°
5π/3
285°
19π/12
270°
3π/2
255°
17π/12
240°
4π/3
225°
5π/4
210°
7π/6
195°
13π/12
π
180°
165°
11π/12
150°
5π/6
135°
3π/4
120°
2π/3
105°
7π/12