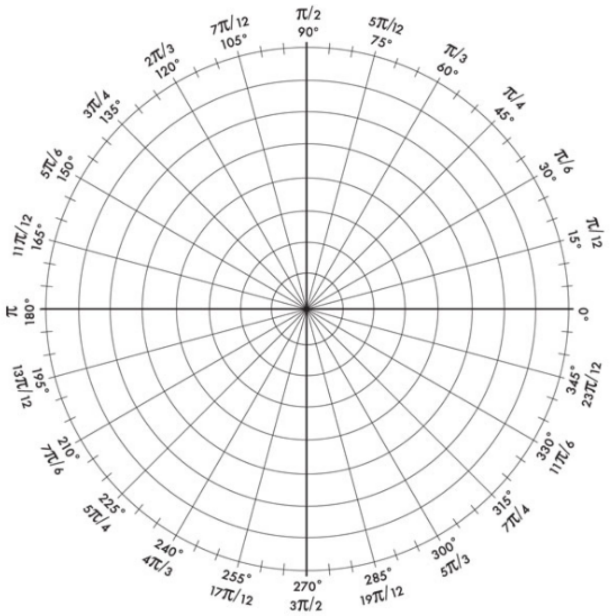
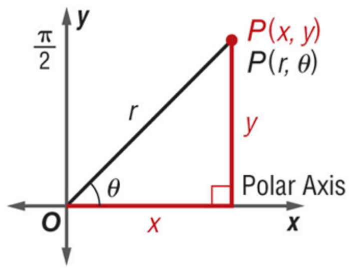

π/2
90°
5π/12
75°
π/3
60°
π/4
45°
π/6
30°
π/12
15°
0°
345°
23π/12
330°
11π/6
315°
7π/4
300°
5π/3
285°
19π/12
270°
3π/2
255°
17π/12
240°
4π/3
225°
5π/4
210°
7π/6
195°
13π/12
180°
π
165°
11π/12
150°
5π/6
135°
3π/4
120°
2π/3
105°
7π/12

KeyConcept Convert Polar to Rectangular Coordinates

If a point P has polar coordinates (r, θ), then the rectangular coordinates (x, y) of P are given by

$x = r\cos\theta$ and $y = r\sin\theta$.

That is, $(x, y) = (r\cos\theta, r\sin\theta)$.

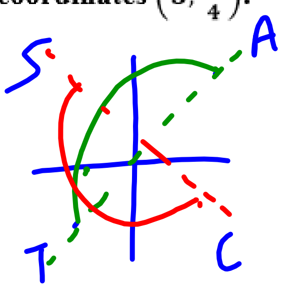


9-3 Study Guide and Intervention

Polar and Rectangular Forms of Equations

Polar and Rectangular Coordinates If a point P has polar coordinates (r, θ), then the rectangular coordinates (x, y) of P are given by $x = r \cos \theta$ and $y = r \sin \theta$. If a point P has rectangular coordinates (x, y), then the polar coordinates (r, θ) of P are given by $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1} \frac{y}{x}$, when $x > 0$, and $\theta = \tan^{-1} \frac{y}{x} + \pi$, when $x < 0$.

Example 1: Find rectangular coordinates for point P with the polar coordinates $\left(3, \frac{3\pi}{4}\right)$.



For $P\left(3, \frac{3\pi}{4}\right)$, $r = 3$ and $\theta = \frac{3\pi}{4}$. Use the conversion formulas.

$x = r \cos \theta$ $\qquad$ $y = r \sin \theta$

$= 3 \cos \frac{3\pi}{4}$ $\qquad$ $= 3 \sin \frac{3\pi}{4}$

$= 3\left(-\frac{\sqrt{2}}{2}\right)$ or $-\frac{3\sqrt{2}}{2}$ $\qquad$ $= 3\left(\frac{\sqrt{2}}{2}\right)$ or $\frac{3\sqrt{2}}{2}$

The rectangular coordinates of P are $\left(-\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$, or $(-2.12, 2.12)$ to the nearest hundredth.

Example 2: Find two pairs of polar coordinates for point R with the rectangular coordinates $(5, -9)$.

For $R(5, -9)$, $x = 5$ and $y = -9$.

$r = \sqrt{x^2 + y^2}$ $\qquad$ $\theta = \tan^{-1} \frac{y}{x}$

$= \sqrt{5^2 + (-9)^2}$ $\qquad$ $= \tan^{-1} \frac{-9}{5}$

$= \sqrt{106}$ or about 10.30 $\qquad$ $= -1.06$

One pair of polar coordinates for R is $(10.30, -1.06)$. To obtain a second pair of polar coordinates for R, you can add 2π to the θ-value. This results in $(10.30, -1.06 + 2\pi)$ or $(10.30, 5.22)$.

Exercises

Find rectangular coordinates for each point with the given polar coordinates.

1. (20, –60°)

$x = r\cos\theta$

$x = 20\cos -60°$

$x = 20\left(\frac{1}{2}\right)$

$x = 10$

$y = r\sin\theta$

$y = 20 \cdot \sin -60°$

$y = 20 \cdot -\frac{\sqrt{3}}{2}$

$y = -10\sqrt{3}$

$(10, -10\sqrt{3})$

4. $\left(3, \frac{\pi}{3}\right)$

$x = 3\cos 60$

$x = 3 \cdot \frac{1}{2}$

$x = \frac{3}{2}$

$y = 3\sin\frac{\pi}{3}$

$y = 3 \cdot \frac{\sqrt{3}}{2}$

$y = \frac{3\sqrt{3}}{2}$

$\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$

Find two pairs of polar coordinates for each point with the given rectangular coordinates if $0 \le \theta \le 2\pi$.

5. (2, –2)

$r = \sqrt{x^2 + y^2}$

$r = \sqrt{(2)^2 + (-2)^2}$

$r = \sqrt{8}$

$r = 2\sqrt{2}$

$\theta = \tan^{-1}\frac{y}{x}$

$\theta = \tan^{-1}\frac{-2}{2}$

$\theta = \tan^{-1} -1$

$\theta = \frac{7\pi}{4}$

$\left(-2\sqrt{2}, \frac{3\pi}{4}\right), \left(2\sqrt{2}, \frac{7\pi}{4}\right)$

6. (3, 5)

$r = \sqrt{3^2 + 5^2}$

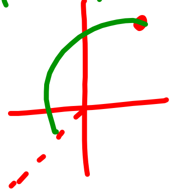
$r = \sqrt{34}$

$\theta = \tan^{-1}\frac{5}{3}$

$\theta = 1.03$

$(\sqrt{34}, 1.03)$

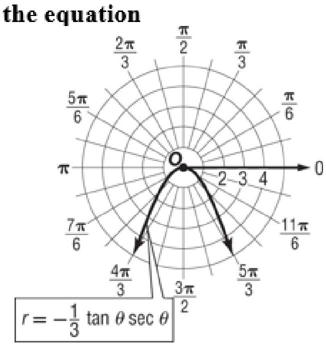
$(-\sqrt{34}, 4.17)$



9-3 Study Guide and Intervention *(continued)*
Polar and Rectangular Forms of Equations

Polar and Rectangular Equations You can also use the relationships $r^2 = x^2 + y^2$, $x = r\cos\theta$ and $y = r\sin\theta$, and $\tan\theta = \frac{y}{x}$ to convert between rectangular equations and polar equations.

Example 1: Identify the graph of the rectangular equation $y = -3x^2$. Then write the equation in polar form. Support your answer by graphing the polar form of the equation.

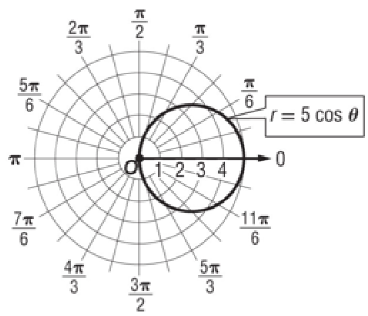


The graph of $y = -3x^2$ is a parabola with vertex at the origin that opens down.

$y = -3x^2$	Original equation
$r\sin\theta = -3(r\cos\theta)^2$	$x = r\cos\theta$ and $y = r\sin\theta$
$r\sin\theta = -3r^2\cos^2\theta$	Multiply.
$\frac{\sin\theta}{-3\cos^2\theta} = r$	Divide each side by $-3r\cos^2\theta$.
$-\frac{1}{3}\tan\theta\sec\theta = r$	Quotient and Reciprocal Identities

The graph of the polar equation $r = -\frac{1}{3}\tan\theta\sec\theta$ is a parabola with vertex at the pole that opens down.

Example 2: Write the polar equation $r = 5\cos\theta$ in rectangular form and then identify its graph. Support your answer by graphing the polar form of the equation.



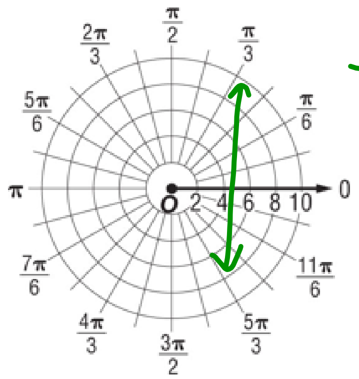
$r = 5\cos\theta$	Original equation
$r^2 = 5r\cos\theta$	Multiply each side by r.
$x^2 + y^2 = 5x$	$r^2 = x^2 + y^2$ and $r\cos\theta = x$
$x^2 - 5x + y^2 = 0$	Subtract $5x$ from each side.

Because in standard form this equation is $(x - 2.5)^2 + y^2 = 6.25$, you can identify the graph of this equation as a circle centered at (2.5, 0) with radius 2.5, as supported by the graph of $r = 5\cos\theta$.

Exercises

Identify the graph of each rectangular equation. Then write the equation in polar form. Support your answer by graphing the polar form of the equation.

1. $x = 5$

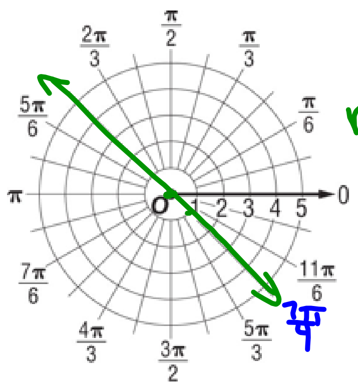


$$x = 5$$

$$\frac{r\cos\theta}{\cos\theta} = \frac{5}{\cos\theta}$$

$$r = \frac{5}{\cos\theta} = 5 \cdot \frac{1}{\cos\theta} = 5\sec\theta$$

2. $y = -x$



$$r\sin\theta = -r\cos\theta$$

$$+r\cos\theta \quad +r\cos\theta$$

$$r\sin\theta + r\cos\theta = 0$$

$$r(\sin\theta + \cos\theta) = 0$$

~~$r = 0$~~

$$\sin\theta + \cos\theta = 0$$

$$-\cos\theta \quad -\cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = \frac{-\cos\theta}{\cos\theta}$$

$$\tan\theta = -1$$

$$\theta = \frac{7\pi}{4}$$

Hw: p. 557, 1, 5, 11, 15, 19, 23, 27, 33, 39, 45, 47