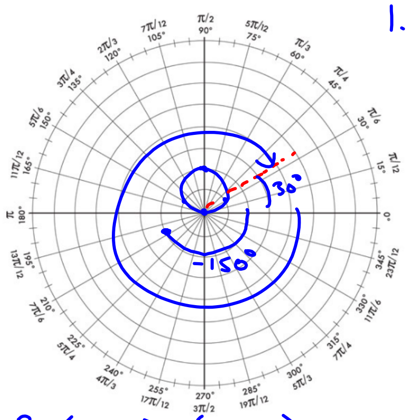


1. $r = 2\sin\theta$

θ	r
$-\frac{\pi}{2}$	-2
$-\frac{\pi}{3}$	$-\sqrt{3}$
$-\frac{\pi}{4}$	$-\sqrt{2}$
$-\frac{\pi}{6}$	-1
0	0
$\frac{\pi}{6}$	1
$\frac{\pi}{4}$	$\sqrt{2}$
$\frac{\pi}{3}$	$\sqrt{3}$
$\frac{\pi}{2}$	2

2. $(2, -150°)$ $(-2, 30°)$

$(2, 210°)$ $(-2, -330°)$

3. $(\overset{r}{-6}, \overset{\theta}{\frac{\pi}{6}})$

$x = r\cos\theta$

$= -6\cos\frac{\pi}{6}$

$= \frac{\cancel{-6}^{-3}}{1}\cdot\frac{\sqrt{3}}{\cancel{2}}$

$x = -3\sqrt{3}$

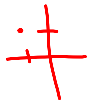
$y = r\sin\theta$

$y = -6\sin\frac{\pi}{6}$

$y = -6\cdot\frac{1}{2}$

$y = -3$

$(-3\sqrt{3}, -3)$



4. $(\overset{x}{-\sqrt{2}}, \overset{y}{\sqrt{2}})$

$r = \sqrt{x^2+y^2}$

$r = \sqrt{(-\sqrt{2})^2+(\sqrt{2})^2}$

$r = \sqrt{2+2}$

$r = \sqrt{4}$

$r = 2$

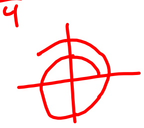
$\theta = \tan^{-1}\frac{y}{x} + \pi$

$\theta = \tan^{-1}\frac{\sqrt{2}}{-\sqrt{2}} + \pi$

$\theta = \tan^{-1} -1 + \pi$

$\theta = \frac{3\pi}{4}$ or $\frac{7\pi}{4} + \pi = \frac{11\pi}{4}$

$(2, \frac{3\pi}{4})$ $(2, \frac{11\pi}{4})$



5. $y = x^2$

$r\sin\theta = (r\cos\theta)^2$

$(r\cos\theta)(r\cos\theta) = r^2 + r\cos\theta \ldots$

$\frac{\cancel{r}\sin\theta}{\cancel{r}} = \frac{r^{\cancel{2}}\cos^2\theta}{\cancel{r}}$

$\frac{\sin\theta}{\cos^2\theta} = \frac{r\cancel{\cos^2\theta}}{\cancel{\cos^2\theta}}$

$r = \frac{\sin\theta}{\cos^2\theta}$

$r = \frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\cos\theta}$

$r = \tan\theta\cdot\sec\theta$

9-5 Study Guide and Intervention

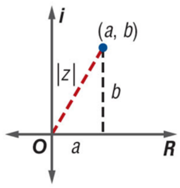
Complex Numbers and ~~De Moivre's Theorem~~

Polar Forms of Complex Numbers You can convert the complex number $z = a + bi$ to polar or trigonometric form using the formula $z = r(\cos\theta + i\sin\theta)$, where $r = |z| = \sqrt{a^2 + b^2}$ and $\theta = \tan^{-1}\frac{b}{a}$ for $a > 0$, $\theta = \tan^{-1}\frac{b}{a} + \pi$ for $a < 0$. You can also convert a complex number in polar form to rectangular form by evaluating it for given values of r and θ using the formulas $a = r\cos\theta$ and $b = r\sin\theta$.

KeyConcept Absolute Value of a Complex Number

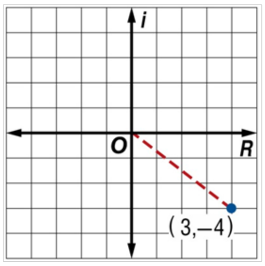
The absolute value of the complex number $z = a + bi$ is

$$|z| = |a + bi| = \sqrt{a^2 + b^2}.$$

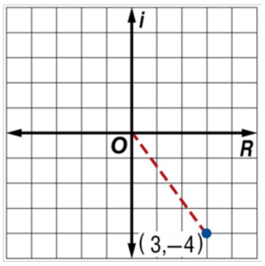


Graph 3 – 4*i* in the complex plane and find its absolute value.

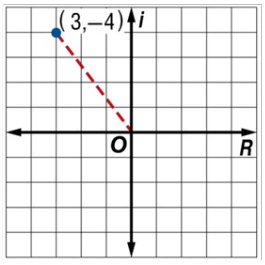
A. 5;



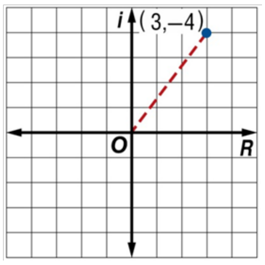
C. 1;



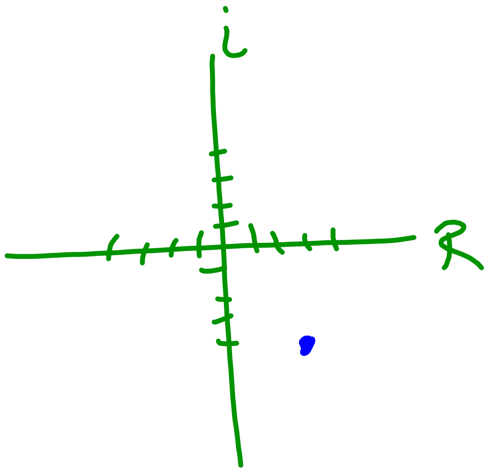
B. 5;



D. 7;



$$|\overset{a}{3} - \overset{b}{4}i| = \sqrt{a^2 + b^2}$$
$$= \sqrt{(3)^2 + (-4)^2}$$
$$= \sqrt{9 + 16}$$
$$= \sqrt{25}$$
$$= 5$$

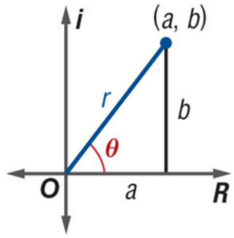


KeyConcept Polar Form of a Complex Number

The polar or trigonometric form of the complex number $z = a + b\mathbf{i}$ is

$$z = r(\cos\theta + \mathbf{i}\sin\theta), \text{ where}$$

$r = |z| = \sqrt{a^2 + b^2}$, $a = r\cos\theta$, $b = r\sin\theta$, and $\theta = \tan^{-1}\frac{b}{a}$ for $a > 0$ or $\theta = \tan^{-1}\frac{b}{a} + \pi$ for $a < 0$.



Example 1: **Express $2\sqrt{3} - 2i$ in polar form.**

First find the modulus.

$r = \sqrt{a^2 + b^2}$

$= \sqrt{(2\sqrt{3})^2 + (-2)^2}$

$= 4$

Then find the argument.

$\theta = \tan^{-1} \frac{b}{a}$

$= \tan^{-1} \frac{2}{2\sqrt{3}}$ or $\frac{11\pi}{6}$

The polar form of $2\sqrt{3} - 2i$ is $4\left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6}\right)$ or about $4(\cos 5.76 + i \sin 5.76)$.

Example 2: **Express $z = 2\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)$ in rectangular form.**

Evaluate the trigonometric values and simplify.

$2\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right) = 2\left(\frac{\sqrt{2}}{2} + i\left(\frac{\sqrt{2}}{2}\right)\right)$ or $\sqrt{2} + i\sqrt{2}$

The rectangular form of z is $\sqrt{2} + i\sqrt{2}$.

Exercises

Express each complex number in polar form.

$a \quad b \qquad + \; i \quad r$

1. $1 - 1i$

$r = \sqrt{a^2 + b^2}$ $\quad \theta = \tan^{-1} \frac{b}{a}$

$r = \sqrt{(1)^2 + (-1)^2}$ $\quad \theta = \tan^{-1} \frac{-1}{1}$

$r = \sqrt{2}$ $\quad \theta = \tan^{-1} -1$

$\theta = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$

$r(\cos\theta + i\sin\theta)$

$\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

2. $3 + 2i$

$r = \sqrt{(3)^2 + (2)^2}$

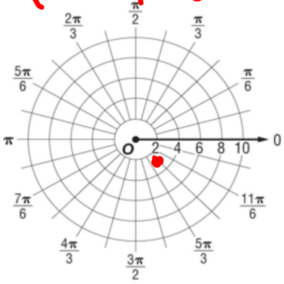
$r = \sqrt{13}$

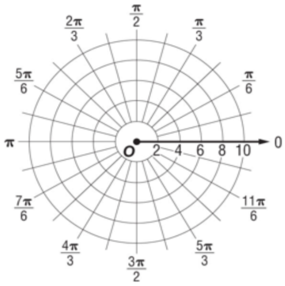
$\theta = \tan^{-1} \frac{2}{3}$

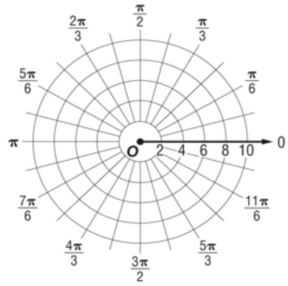
$\theta = .58$

$z = \sqrt{13}\,(\cos .58 + i\sin .58)$

3. $-1 + \sqrt{3}i$

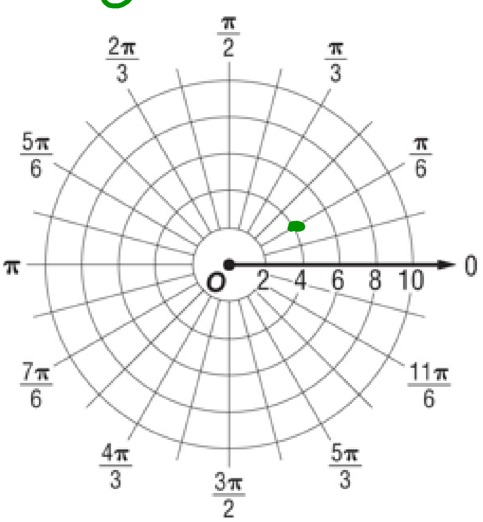






4. $4\left(\cos\frac{\pi}{6} + \boldsymbol{i}\sin\frac{\pi}{6}\right)$

r θ



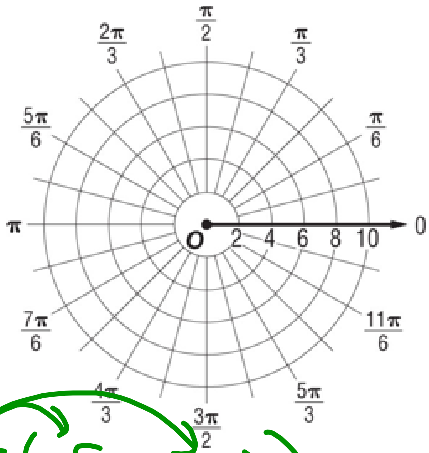
4-6 Graph on polar grid and find rectangular form.

$4\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$4\left(\frac{\sqrt{3}}{2} + i\cdot\frac{1}{2}\right)$

$\boxed{2\sqrt{3} + 2i}$

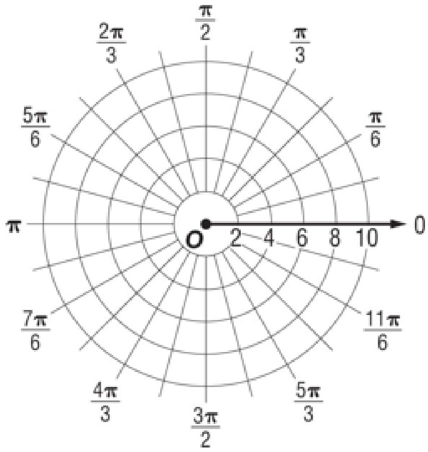
5. $4\sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$



$4\sqrt{2}\left(-\frac{\sqrt{2}}{2} + i\left(-\frac{\sqrt{2}}{2}\right)\right)$

$-4-4i$

6. $6\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$



Products, Quotients, Powers, and Roots of Complex Numbers Use these formulas to multiply and divide complex numbers.

Given the complex numbers $z_1 = r_1(\cos\theta_1 + \boldsymbol{i}\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + \boldsymbol{i}\sin\theta_2)$,

Product Formula: $z_1 z_2 = r_1 r_2\,[\cos(\theta_1 + \theta_2) + \boldsymbol{i}\sin(\theta_1 + \theta_2)]$

Quotient Formula: $\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + \boldsymbol{i}\sin(\theta_1 - \theta_2)]$

Example: Find $3\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\cdot 4\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$, and express it in rectangular form.

$3\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\cdot 4\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$	Original expression
$= 3(4)\left[\cos\left(\frac{\pi}{4} + \frac{\pi}{2}\right) + \boldsymbol{i}\sin\left(\frac{\pi}{4} + \frac{\pi}{2}\right)\right]$	Product Formula
$= 12\left(\cos\frac{3\pi}{4} + \boldsymbol{i}\sin\frac{3\pi}{4}\right)$	Simplify.
$= 12\left(-\frac{\sqrt{2}}{2} + \boldsymbol{i}\frac{\sqrt{2}}{2}\right)$	Evaluate.
$= -6\sqrt{2} + 6\boldsymbol{i}\sqrt{2}$	Distributive Property

The polar form of the product is $12\left(\cos\frac{3\pi}{4} + \boldsymbol{i}\sin\frac{3\pi}{4}\right)$ and the rectangular form of the product is $-6\sqrt{2} + 6\boldsymbol{i}\sqrt{2}$.

Products, Quotients, Powers, and Roots of Complex Numbers Use these formulas to multiply and divide complex numbers.

Given the complex numbers $z_1 = r_1(\cos \theta_1 + \boldsymbol{i} \sin \theta_1)$ and $z_2 = r_2(\cos \theta_2 + \boldsymbol{i} \sin \theta_2)$,

Product Formula: $z_1 z_2 = r_1 r_2 \left[\cos(\theta_1 + \theta_2) + \boldsymbol{i} \sin(\theta_1 + \theta_2)\right]$

Quotient Formula: $\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + \boldsymbol{i} \sin(\theta_1 - \theta_2)\right]$

Find each product or quotient and express it in rectangular form.

1. $\overset{r_1}{3}\left(\cos \overset{\theta_1}{\frac{\pi}{3}} + \boldsymbol{i} \sin \frac{\pi}{3}\right) \cdot \overset{r_2}{3}\left(\cos \overset{\theta_2}{\frac{5\pi}{3}} + \boldsymbol{i} \sin \frac{5\pi}{3}\right)$

$(3)(3)\left[\cos\left(\frac{\pi}{3}+\frac{5\pi}{3}\right) + i \sin\left(\frac{\pi}{3}+\frac{5\pi}{3}\right)\right]$

$9(\cos 2\pi + i \sin 2\pi)$

$9(1 + i(0))$

$9(1)$

9

Products, Quotients, Powers, and Roots of Complex Numbers Use these formulas to multiply and divide complex numbers.

Given the complex numbers $z_1 = r_1(\cos\theta_1 + \boldsymbol{i}\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + \boldsymbol{i}\sin\theta_2)$,

Product Formula: $z_1 z_2 = r_1 r_2\,[\cos(\theta_1 + \theta_2) + \boldsymbol{i}\sin(\theta_1 + \theta_2)]$

Quotient Formula: $\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + \boldsymbol{i}\sin(\theta_1 - \theta_2)]$

2. $\underbrace{6}_{r_1}\left(\cos\overbrace{\frac{\pi}{2}}^{\theta_1} + \boldsymbol{i}\sin\frac{\pi}{2}\right) \div \underbrace{2}_{r_2}\left(\cos\overbrace{\frac{\pi}{3}}^{\theta_2} + \boldsymbol{i}\sin\frac{\pi}{3}\right)$

$$\frac{z_1}{z_2} = \frac{6}{2}\left[\cos\left(\frac{\pi}{2}-\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{2}-\frac{\pi}{3}\right)\right]$$

$$= 3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

$$= 3\left(\frac{\sqrt{3}}{2} + i\cdot\frac{1}{2}\right)$$

$$= \frac{3\sqrt{3}}{2} + \frac{3}{2}i$$

Hw: p. 577

1, 5, 9, 11, 15, 19, 21, 27, 31, 35