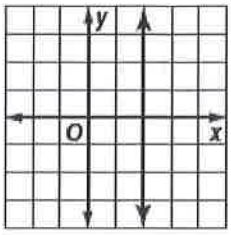
Name: Key

Semester 1 Test Review

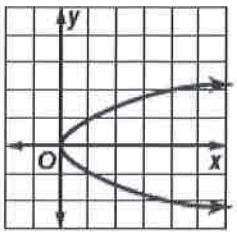
Write the letter for the correct answer in the blank at the right of each question.

1. Which relation is a function?

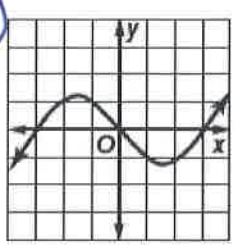
a.



b.

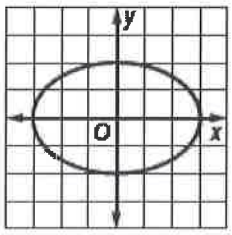


c.



passes vertical line test

d.



2. Find the zero of $f(x) = -\frac{2}{3}x - 12$.

a. –18
b. –12
c. 12
d. 18

$0 = -\frac{2}{3}x - 12$

$+12 \qquad +12$

$-\frac{3}{2}(12) = \left(-\frac{2}{3}x\right) -\frac{3}{2}$

$-18 = x$

3. What is $f(-2)$ for $f(x) = \begin{cases} |4x| \text{ if } x < -2 \\ x^3 - 1 \text{ if } x \geq -2 \end{cases}$?

a. 8
b. –7
c. –8
d. –9

$f(-2) = (-2)^3 - 1$

$= -8 - 1$

$= -9$

4. Which function is an odd function?

a. $f(x) = -x^3 + 4$

b. $f(x) = 2x^3$

c. $f(x) = x^4 - 9$

d. $f(x) = x^4 + 4x$

$f(-x) = -(-x)^3 + 4 = x^3 + 4$ ✗ not opposite of $f(x)$

$f(-x) = 2(-x)^3 = -2x^3$ ✓ opposite of $f(x)$

$f(-x) = (-x)^4 - 9 = x^4 - 9$ ✗ not opposite of $f(x)$

$f(-x) = (-x)^4 + 4(-x) = x^4 - 4x$ ✗ not opposite of $f(x)$

5. Which function has a hole?

a. $f(x) = \dfrac{x}{x+3}$

b. $f(x) = \dfrac{x^2-4}{x+2}$

c. $f(x) = \dfrac{1}{x+3}$

d. $f(x) = x^3 - 3$

$f(x) = \dfrac{x^2-4}{x+2} = \dfrac{(x+2)(x-2)}{x+2}$ → common factor ⇒ hole that cancels

6. For $f(x) = \dfrac{3}{x-4}$, what expression for a makes $\lim_{x \to \infty} f(x) = a$ correct?

a. $-\infty$

b. 0

c. 3

d. ∞

horizontal asymptote at $y = 0$ since degree of numerator isn't larger than denominator.

7. **TEAMS** The function $p(x) = -5x^3 + 47x^2 - 109x + 90$ approximates the number of students on the debate team from 2004 to 2010 where x is the number of years since 2000. Which of the following best approximates the relative maximum of the function?

a. 5

b. 15

c. 97

d. 10,880

x	0	1	2	3	4	5	6	7	8	9	10
$p(x)$	90	23	20	51	86	95	48	−85	−334	−729	−1300

↑ ≈ relative max

8. **ROCKETS** The height in feet of a rocket t seconds after launch is modeled by $h(t) = -16t^2 + 72t$. Find its average speed from 3 to 4 seconds.

a. –56 ft/sec

b. –40 ft/sec

c. 40 ft/sec

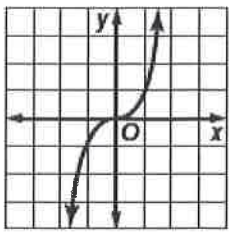
d. 56 ft/sec

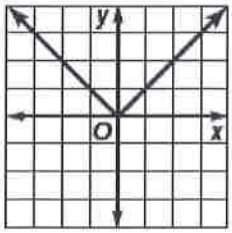
$h(3) = -16(3)^2 + 72(3) = 72$

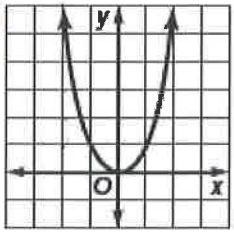
$h(4) = -16(4)^2 + 72(4) = 32$

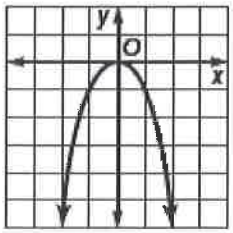
$m = \dfrac{72 - 32}{3 - 4}$

9. Which of the following represents the graph of $f(x) = |x^3|$?

a. 

b. 

c. 

d. 

unique point at $x=-2$

$f(-2) = |(-2)^3|$

$= |-8|$

$= 8$

$(-2, 8)$

10. If $f(x) = x - 3$ and $g(x) = 2x - 4$, find $(f+g)(x)$.

a. $(f+g)(x) = 3x - 7$

b. $(f+g)(x) = -x - 7$

c. $(f+g)(x) = -x + 1$

d. $(f+g)(x) = 3x + 1$

$f(x) + g(x)$

$(x-3) + (2x-4)$

$3x - 7$

11. If $f(x) = x^2 + 1$ and $g(x) = 2x$, find $[f \circ g](x)$.

a. $[f \circ g](x) = 2x^2 + 2$

b. $[f \circ g](x) = 2x^2 + 1$

c. $[f \circ g](x) = x^2 + 4x + 4$

d. $[f \circ g](x) = 4x^2 + 1$

$f(g(x)) = (2x)^2 + 1$

$= 4x^2 + 1$

12. Find the inverse of $f(x) = 2x + 9$.

a. $f^{-1}(x) = -2x - 9$

b. $f^{-1}(x) = -9 - 2x$

c. $f^{-1}(x) = \frac{x-9}{2}$

d. $f^{-1}(x) = \frac{9-x}{2}$

$y = 2x + 9$

$x = 2y + 9$

$-9 \quad -9$

$\frac{x-9}{2} = \frac{2y}{2}$

$f^{-1}(x) = \frac{x-9}{2}$

13. Which function has an inverse that is also a function?

a. $f(x) = |x|$ — x

b. $f(x) = x^2$ — x

c. $f(x) = \frac{x+1}{x^2}$ — x

d. $f(x) = x^3$ passes horizontal line test, cubic parent function

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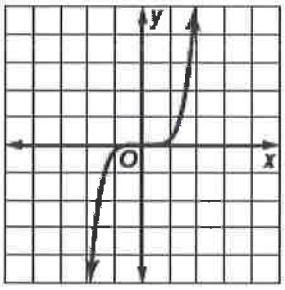
14. The graph of $f(x) = \frac{1}{10}x^5$ is shown at the right. What is the domain of the function?



a. $(-\infty, \infty)$

b. $(0, \infty)$

c. $(-\infty, 0)$ or $(0, \infty)$

d. $(-\infty, 0)$

15. Which describes the end behavior of the graph of $f(x) = 2x^3 - 5x + 1$?

a. $\lim_{x \to -\infty} f(x) = \infty$, $\lim_{x \to \infty} f(x) = \infty$

b. $\lim_{x \to -\infty} f(x) = -\infty$, $\lim_{x \to \infty} f(x) = -\infty$

c. $\lim_{x \to -\infty} f(x) = -\infty$, $\lim_{x \to \infty} f(x) = \infty$

d. $\lim_{x \to -\infty} f(x) = \infty$, $\lim_{x \to \infty} f(x) = -\infty$

Since the leading coefficient is positive, right end behavior is ∞. Since degree is odd, left end behavior will be the opposite, $-\infty$.

16. What is the greatest possible number of real zeros of $f(x) = x^3 - 2x^2 - x + 1$?

a. 1

b. 2

c. 3

d. 4

$n = 3$ (degree)

17. What is the greatest possible number of turning points of $f(x) = 6x^4 + 11x^3 - x^2 + x$?

a. 3
b. 4
c. 5
d. 6

# of turning points = $n-1$
$= 4-1$
$= 3$

18. Divide $(x^3 + 5x^2 + 5x - 2)$ by $(x + 2)$ using synthetic division.

a. $x^2 + 7x + 19 + \frac{36}{x+2}$
b. $x^2 + 4$
c. $x^2 + 3x - 1$
d. $x^2 + 7x - 9 + \frac{16}{x+2}$

-2 | 1 5 5 -2
↓ -2 -6 2
1 3 -1 0
$x^2 + 3x - 1$

19. **PHYSICS** The height h in feet of a ball thrown into the air after t seconds is given by $h(t) = -16t^2 + 35t + 6$. Use synthetic substitution to find the height of the ball after 0.5 second.

a. 19.5 ft
b. 21 ft
c. 25 ft
d. 26.5 ft

.5 | -16 35 6
↓ -8 13.5
-16 27 19.5
remainder = $h(.5)$

20. Find the remainder when $2x^3 + 6x^2 + 3x - 1$ is divided by $x - 1$. Is the binomial a factor of the polynomial?

a. 0; yes
b. –2; no
c. 10; no
d. –1; yes

1 | 2 6 3 -1
↓ 2 8 11
2 8 11 10

Since the remainder $\neq 0$, $x-1$ is not a factor according to the factor theorem.

21. **FINANCE** For a period of x days, an account balance can be modeled by $f(x) = x^3 - x^2 - 8x$. When was the balance \$60?

a. Day 5
b. Day 8
c. Day 9
d. Day 10

$60 = x^3 - x^2 - 8x$

$(5)^3 - (5)^2 - 8(5)$
$= 125 - 25 - 40$
$= 60$ ✓

$(8)^3 - (8)^2 - 8(8)$
$512 - 64 - 64$
$= 384x$

$(9)^3 - (9)^2 - 8(9)$
$729 - 81 - 72$
$= 576x$

$(10)^3 - (10)^2 - 8(10)$
$1000 - 100 - 80$
$= 820x$

22. What are the vertical asymptotes of $f(x) = \frac{x^2 - 4}{x^2 - 9}$?

a. $x = 0$
b. $x = 1$
c. $x = \pm 2$
d. $x = \pm 3$

$x^2 - 9 = 0$
$+9 \quad +9$
$\sqrt{x^2} = \sqrt{9}$
$x = \pm 3$

23. **MEDICINE** The concentration of a medicine is modeled by $f(x) = \frac{2x}{3x^2 + 1}$. What is the horizontal asymptote of the function?

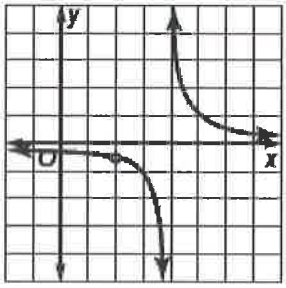
a. $y = -\frac{1}{3}$

(b) $y = 0$ denominator's degree is larger than numerator's degree.

c. $y = \frac{2}{3}$

d. $y = 2$

24. Which of the following could be the function represented by the graph?



hole at $x = 2$

vertical asymptote at $x = 4$

a. $f(x) = \frac{1}{x-4}$

b. $f(x) = \frac{x+2}{x-4}$

(c) $f(x) = \frac{x-2}{(x-2)(x-4)}$ $x-2$ factors cancel mean hole at $x=2$; $x-4=0 \Rightarrow x=4$ for vertical asymptote

d. $f(x) = \frac{x-2}{(x-2)(x+4)}$

25. Solve $\frac{1}{x+4} = \frac{1}{x^2+3x-4} + \frac{4}{x-1}$.

$\left(\frac{1}{x+4}\right)(x+4)(x-1) = \frac{1}{(x+4)(x-1)}(x+4)(x-1) + \frac{4}{x-1}(x+4)(x-1)$ LCD $(x+4)(x-1)$

$x - 1 = 1 + 4(x+4)$

$x - 1 = 4x + 17$

$-x \quad -17 \quad -x \quad -17$

$\frac{-18}{3} = \frac{3x}{3}$ $x = -6$

a. –6

(b) –2

c. 2

d. 6

Check

$\frac{1}{-6+4} = \frac{1}{(-6)^2+3(-6)-4} + \frac{4}{-6-1}$

$-\frac{1}{2} = -\frac{1}{2}$ ✓

26. Which of the following is the solution to $(x+3)(x-2) \leq 0$?

$x+3=0 \quad x-2=0$

$-3 \quad -3 \quad +2 \quad +2$

$x=-3 \quad x=2$

$x=-4 \quad x=0 \quad x=3$

$+ \quad -3 \quad - \quad 2 \quad +$

a. $(-\infty, \infty)$

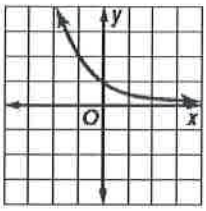
b. $(-\infty, 3)$ or $(2, \infty)$

c. $(-2, 3)$

(d) $[-3, 2]$

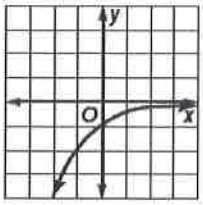
Since ≤ 0, we want the negative interval.

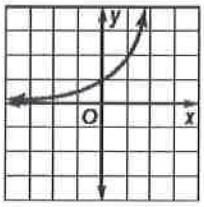
27. Choose the graph of $f(x) = 2^{-x}$.

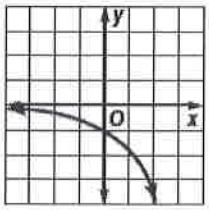
a. 

y-int = (0,1)
exponential decay

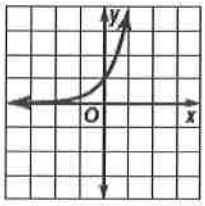
x	-2	-1	0	1	2
f(x)	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$

b. 

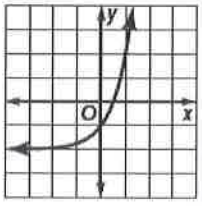
c. 

d. 

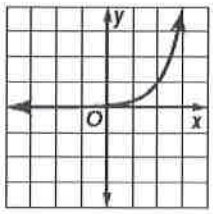
28. Choose the graph of $f(x) = 4^{x-2}$.

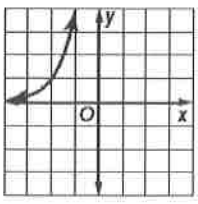
a. 

(2,1) because of horizontal translation right 2

b. 

x	-2	-1	0	1	2
f(x)	$\frac{1}{256}$	$\frac{1}{64}$	$\frac{1}{16}$	$\frac{1}{4}$	1

c. 

d. 

29. Evaluate $\log_9 \frac{1}{27}$.

a. $\frac{2}{3}$

b. $\frac{3}{2}$

c. $-\frac{2}{3}$

d. $-\frac{3}{2}$

$9^x = \frac{1}{27}$

$3^{2x} = 3^{-3}$

$\frac{2x}{2} = \frac{-3}{2}$

$x = -\frac{3}{2}$

30. Solve $\log_4 x + \log_4 (x-2) = \log_4 15$.

a. –3 only

b. 5 only

c. –3 or 5

d. –5 or 3

$\log_4 x + \log_4 (x-2) = \log_4 15$

$x(x-2) = 15$

$-15 \quad -15$

$x^2 - 2x - 15 = 0$

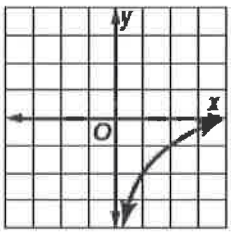
$(x-5)(x+3) = 0$

$x-5=0 \qquad x+3=0$

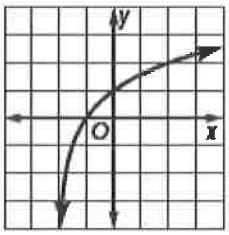
$x=5 \qquad x=-3$ extraneous

31. Choose the graph of $f(x) = \log_2 (x+2)$.

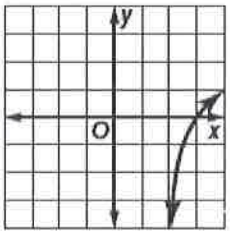
a.



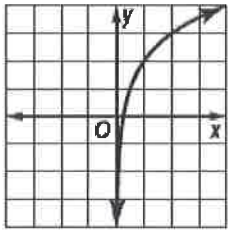
b.



c.



d.



Find inverse

$x = \log_2 (y+2)$

$2^x = y+2$

$-2 \quad -2$

$2^x - 2 = y$

reverse $x+y$ when we graph

x	-2	-1	0	1	2
y	$-1\frac{3}{4}$	$-1\frac{1}{2}$	-1	0	2

32. Solve $5^x = 3^{x+2}$.

a. 2.732
b. 3.109
c. 4.117
d. 4.301

$\ln(5^x) = \ln(3^{x+2})$

$x \ln 5 = (x+2)\ln 3$

$x \ln 5 = x \ln 3 + 2 \ln 3$

$-x \ln 3 \quad -x \ln 3$

$x \ln 5 - x \ln 3 = 2 \ln 3$

$x(\ln 5 - \ln 3) = \frac{2 \ln 3}{\ln 5 - \ln 3}$

$x = 4.301$

33. Convert $\log_5 47$ to a natural logarithm and evaluate.

a. 0.770
b. 2.241
c. 2.392
d. 2.516

$5^x = 47$

$\ln 5^x = \ln 47$

$\frac{x \ln 5}{\ln 5} = \frac{\ln 47}{\ln 5}$

$x = 2.392$

34. Solve $e^{0.2x} = 21.2$.

a. –1.898
b. 4.663
c. 8.234
d. 15.270

$\ln(e^{.2x}) = \ln(21.2)$

$\frac{.2x}{.2} = \frac{\ln 21.2}{.2}$

$x = 15.270$

35. **BANKING** Find the amount of time required for an investment to double at a rate of 12.3% if the interest is compounded continuously.

a. 5.635 years
b. 6.241 years
c. 7.770 years
d. 8.325 years

$A = Pe^{rt}$

$\frac{2P}{P} = \frac{Pe^{.123t}}{P}$

$\ln(2) = \ln(e^{.123t})$

$\frac{.123t}{.123} = \frac{\ln 2}{.123}$

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36. Let $\tan\theta = \frac{12}{5}$, where $\sin\theta > 0$. Find the exact value of $\sin\theta$.

a. $\frac{5}{13}$
b. $\frac{5}{12}$
c. $\frac{12}{13}$
d. $\frac{13}{12}$